ФИЗИКА

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МАГНИТНЫЙ ПОТОК КОЛЬЦА С ТОКОМ. ИНДУКТИВНОСТЬ КОЛЬЦА

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MAGNETIC FLUX OF RING WITH CURRENT. RING INDUCTANCE

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Аннотация. В литературе по физике, посвященной разделу «Электричество и магнетизм», рассматриваются различные формы проводников с токами, создающих магнитное поле. Одним из них является кольцо. Кольцо или одновитковый контур является элементом многих электрических устройств. Одним их основных параметров кольца является индуктивность. В данной работе вначале рассматривается магнитный поток, создаваемый кольцом с током. Затем по определенному магнитному потоку находится индуктивность кольца. Полученная формула находится в хорошем соответствии с другими выражениями для индуктивности кольца.

Ключевые слова: *кольцо с током*, *индукция магнитного поля*, *магнитный поток*, *индуктивность*.

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Abstract. In the literature in physics, devoted to the section “Electricity and magnetism”, various forms of conductors with currents that create a magnetic field are considered. One of them is the ring. A ring or a single-turn circuit is an element of many electrical devices. One of the main parameters of the ring is inductance. In this work, we first consider the magnetic flux created by a current-carrying ring. Then, based on a certain magnetic flux, the inductance of the ring is found. The resulting formula is in good agreement with the other expressions for the inductance of the ring.

Keywords: *current ring*, *magnetic field induction*, *magnetic flux*, *inductance*.

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Introduction

The study of the inductance properties of current-carrying circuits began to actively develop after the invention of the radio. Nowadays, inductance is needed by a huge number of different customers with a huge range of geometric sizes, small of which have hundreds of microns or fewer. One of the simplest elements of electrical circuits is a ring or a single-turn circuit. To determine the inductance of the ring, formulas from [1]–[3], which are approximate, are often used.

At the same time, it is of great interest to obtain an expression for inductance directly from the definition. In this case, inductance is the coefficient of proportionality between the current and the magnetic flux through the surface limited by the current-carrying circuit [4]–[10]

$$\Phi = LI, \qquad (0.1)$$

where Φ – is the magnetic flux, L – is the inductance of the ring, I – is the conductor current.

1 Determination of magnetic field induction inside the ring

The magnetic field induction of a current system can be determined by using the Biot – Savart – Laplace law [4]–[10]. Finding the magnetic field induction of a circular current within the framework of a general physics course is presented, as a rule, only for points lying on the axis of the coil. In this work, we consider the possibility of using the Biot – Savart – Laplace law for points inside the coil with current (in the plane of the coil).

Let us consider a circular current of radius R and a current element $I\,d\vec{l}$ (Figure 1.1).

Let us find the magnetic field induction of the current element at a point located at a distance ρ (|ON|) from the center of the circle. According to the Biot – Savart – Laplace law, the magnetic induction created by a current element

$$d\vec{B} = \frac{\mu_0 I\left[d\vec{l} \times \vec{r}\right]}{4\pi r^3}, \qquad (1.1)$$

where μ_0 – is the magnetic constant, I – is the magnitude of the current, the vector $\vec{r}$ is directed from the current element to the point under study.

The magnitude of magnetic induction taking into account relation (1.1)

$$dB = \frac{\mu_0 I dl}{4\pi r^2}\sin\alpha, \qquad (1.2)$$

where α – is the angle between the vectors $d\vec{l}$ and $\vec{r}$.

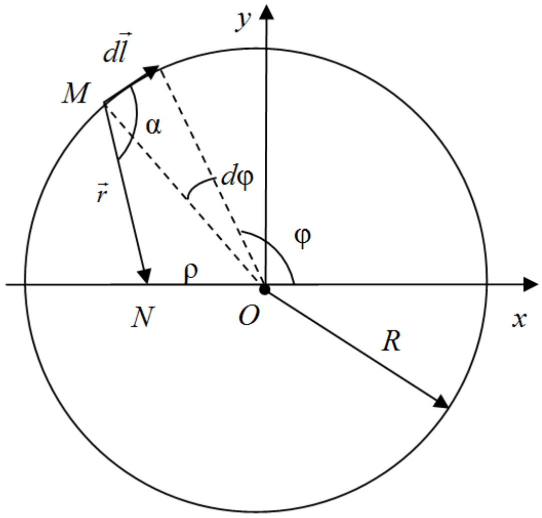

Figure 1.1

The vector $d\vec{B}$ in this case will be directed beyond the plane of the drawing.

Considering that $dl = Rd\varphi$, expression (1.2) can be represented in the form

$$dB = \frac{\mu_0 IRd\varphi}{4\pi r^2}\sin\alpha. \qquad (1.3)$$

Let us find the connection between the angles α and φ. Let the angle φ lie in the range from 0 to π / 2. From Figure 1.2 it follows that

$$r\cos(\alpha - \pi/2) = R + \rho\cos\varphi.$$

Then after transformations we obtain

$$r\sin\alpha - \rho\cos\varphi = R.$$

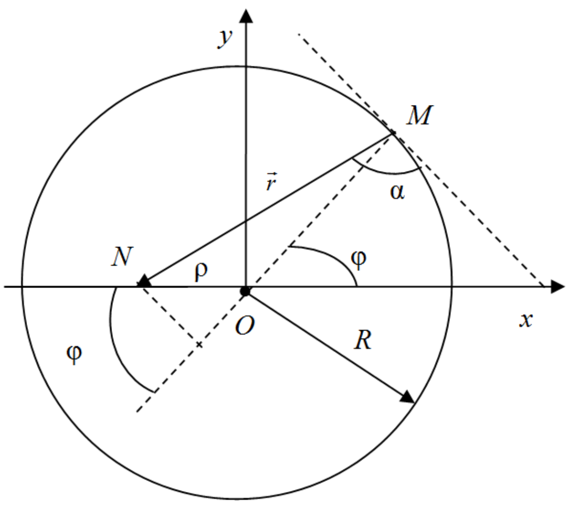

Figure 1.2

Now let the angle φ lie in the range from π / 2 to π. From Figure 1.3 it follows that

$$r\cos(\alpha - \pi/2) + \rho\cos(\pi - \varphi) = R.$$

Then after transformations we obtain

$$r\sin\alpha - \rho\cos\varphi = R.$$

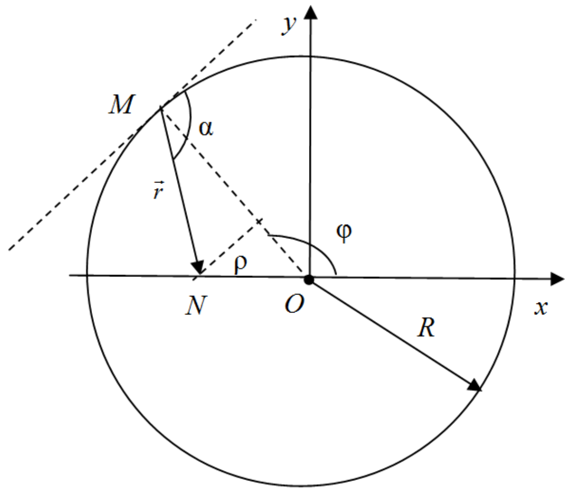

Figure 1.3

Thus, for angles φ lying in the range from 0 to π, the relation is satisfied

$$\sin\alpha = \frac{R + \rho\cos\varphi}{r}. \qquad (1.4)$$

Let's substitute relation (1.4) into expression (1.3)

$$dB = \frac{\mu_0 IR(R + \rho\cos\varphi)d\varphi}{4\pi r^3}. \qquad (1.5)$$

Using the cosine theorem (Figures 1.2 and 1.3) for triangle *ONM* and angle (π – φ), we can write

$$r^2 = \rho^2 + R^2 - 2\rho R\cos(\pi - \varphi)$$

or

$$r = \sqrt{\rho^2 + R^2 + 2\rho R\cos\varphi}. \qquad (1.6)$$

Taking (1.6) into account, expression (1.5) is transformed to the form

$$dB = \frac{\mu_0 IR(R + \rho\cos\varphi)d\varphi}{4\pi\left(\rho^2 + R^2 + 2\rho R\cos\varphi\right)^{1,5}}. \qquad (1.7)$$

Integrating over the entire length of the turn, from (1.7) we obtain

$$B = 2\int_0^{\pi} \frac{\mu_0 IR(R + \rho\cos\varphi)d\varphi}{4\pi\left(\rho^2 + R^2 + 2\rho R\cos\varphi\right)^{1,5}}. \qquad (1.8)$$

In expression (1.8), the two in front of the integral also takes into account the angles φ lying in the range from π to 2π (due to symmetry).

2 Determination of magnetic flux through a surface bounded by a ring

Let us find the elementary magnetic flux through a ring, the inner radius of which is ρ, and the outer radius is ρ + dρ (the width of such a ring is dρ, the length is 2πρ, the area is 2πρdρ, and the vector $d\vec{B}$ is perpendicular to the area *dS*).

$$d\Phi = BdS = B2\pi\rho d\rho. \qquad (2.1)$$

Let us substitute the magnetic induction from (1.8) into expression (2.1)

$$d\Phi = 4\pi\rho\int_0^{\pi}\frac{\mu_0 IR(R+\rho\cos\varphi)d\varphi}{4\pi\left(\rho^2+R^2+2\rho R\cos\varphi\right)^{1,5}}d\rho. \quad (2.2)$$

To find the entire magnetic flux, you need to integrate expression (2.2) over ρ in the range from 0 to R (idealization of the problem when the cross-section of the wire is taken equal to zero).

$$\Phi = \int_0^R\int_0^{\pi}\frac{\mu_0 IR\rho(R+\rho\cos\varphi)}{\left(\rho^2+R^2+2\rho R\cos\varphi\right)^{1,5}}d\varphi d\rho. \quad (2.3)$$

Let us introduce the parameter $k=\rho/R$, then $d\rho = Rdk$ and relation (2.3) will be rewritten as a double integral

$$\Phi = \mu_0 IR\int_0^1\int_0^{\pi}\frac{k(1+k\cos\varphi)}{\left(k^2+1+2k\cos\varphi\right)^{1,5}}d\varphi dk. \quad (2.4)$$

Using (0.1) we write

$$L = \mu_0 R\int_0^1\int_0^{\pi}\frac{k(1+k\cos\varphi)}{\left(k^2+1+2k\cos\varphi\right)^{1,5}}d\varphi dk. \quad (2.5)$$

The integral in (2.5) is an exact solution of the problem under the assumption that the cross section of the conductor is taken equal to zero. However, with such an idealization of the problem, the integral in (2.5) is divergent. In reality, it is necessary to take into account that the conductor itself through which the current flows has a certain diameter d (that is, the cross-section of the conductor is not zero). In this case, expression (2.5) will be presented in the form

$$L = \mu_0 R\int_0^{1-\frac{d}{D}}\int_0^{\pi}\frac{k(1+k\cos\varphi)}{\left(k^2+1+2k\cos\varphi\right)^{1,5}}d\varphi dk, \quad (2.6)$$

where d – is the diameter of the conductor with a round cross-section, D – is the diameter of the ring ($D = 2R$).

Expression (2.6) defines the so-called external inductance and does not take into account the internal inductance caused by the magnetic field inside the conductor. Internal inductance can be neglected if the current flows along the surface of the conductor. This occurs with high-frequency alternating current and a sharp manifestation of the surface effect.

3 Calculation of ring inductance

Let us calculate the inductance using formula (2.6). To do this you need to calculate the double integral

$$J = \int_0^{1-\frac{d}{D}}\int_0^{\pi}\frac{k(1+k\cos\varphi)}{\left(k^2+1+2k\cos\varphi\right)^{1,5}}d\varphi dk \quad (3.1)$$

for a number of values d/D. The calculated values of the integral J (3.1) for some values d/D are presented in Table 3.1 (from formula (2.6) $J = L/\mu_0 R$).

For comparison, we also calculate the induc-tance of the ring using the formulas from [1, p. 207–208]

$$L = \mu_0 R\left(\left(1-\frac{r^2}{4R^2}\ln\frac{8R}{r}+\frac{r^2}{2R^2}\right)\ln\frac{8R}{r}-2+\frac{r^2}{16R^2}\right), \quad (3.2)$$

$$L = \mu_0 R\left(\ln\frac{8R}{r}-2\right). \quad (3.3)$$

Formula (3.3) is a simplification of formula (3.2) for $R \gg r$.

Formulas (3.2) and (3.3), taking into account the relation d/D, will be rewritten in the form

$$L = \mu_0 R\times$$
$$\times\left(\left(1-\frac{d^2}{4D^2}\ln\frac{8D}{d}+\frac{d^2}{2D^2}\right)\ln\frac{8D}{d}-2+\frac{d^2}{16D^2}\right), \quad (3.4)$$

$$L = \mu_0 R\left(\ln\frac{8D}{d}-2\right). \quad (3.5)$$

We will enter the calculation results in Table 3.1. The third and fourth columns contain the results of calculations carried out using formulas (3.4) and (3.5), respectively.

Table 3.1

d/D	J, (3.1)	$L/\mu_0 R$, (3.4)	$L/\mu_0 R$, (3.5)
0,1	2,2177	2,355306	2,382026
0,09	2,33437	2,464278	2,487387
0,08	2,46409	2,585575	2,60517
0,07	2,61028	2,722497	2,738702
0,06	2,77788	2,879888	2,892852
0,05	2,97458	3,065263	3,075174
0,04	3,21319	3,291227	3,298317
0,03	3,5177	3,581436	3,585999
0,02	3,94178	3,989048	3,991465
0,01	4,65626	4,683823	4,684612
0,001	6,9832	6,987181	6,987197

The discussion of the results

Analyzing the dependence of the values of the integral $J(d/D)$, we can note the following.

Firstly, as the ratio d/D increases, the value of the integral $J(d/D)$ decreases. This reduces the inductance of the ring.

Secondly, when the ratio d/D decreases by 10 times (from 0.1 to 0.01), the inductance of the ring does not increase so significantly, a little more than twice.

Thirdly, from table 3.1. it follows that the results of the calculations using formulas (3.4) and (3.5) are in good agreement with the results from formula (3.1). Moreover, the smaller the ratio d/D, the better the fit. At $d/D = 0{,}1$ the error is approximately 6%. At $d/D = 0{,}001$ the error is approximately 0,06%.

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