МАТЕМАТИКА

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ГЛОБАЛЬНАЯ ТЕОРЕМА КОРРЕКТНОСТИ ПЕРВОЙ СМЕШАННОЙ ЗАДАЧИ ДЛЯ ОБЩЕГО ТЕЛЕГРАФНОГО УРАВНЕНИЯ С ПЕРЕМЕННЫМИ КОЭФФИЦИЕНТАМИ НА ОТРЕЗКЕ

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GLOBAL CORRECTNESS THEOREM TO THE FIRST MIXED PROBLEM FOR THE GENERAL TELEGRAPH EQUATION WITH VARIABLE COEFFICIENTS ON A SEGMENT

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Аннотация. Глобальная теорема корректности по Адамару первой смешанной задачи для неоднородного общего телеграфного уравнения со всеми переменными коэффициентами в полуполосе плоскости доказана новым методом вспомогательных смешанных задач. Без явных продолжений данных смешанной задачи за пределы множества их задания выведены рекуррентные формулы типа Римана единственного и устойчивого классического решения для первой смешанной задачи на отрезке. Эта полуполоса плоскости разделена криволинейными характеристиками телеграфного уравнения на прямоугольники одинаковой высоты, а каждый прямоугольник – на три треугольника. Критерий корректности состоит из требований гладкости и условий согласования на правые части уравнения, начальных и граничных условий смешанной задачи. Требования гладкости необходимы и достаточны для дважды непрерывной дифференцируемости решения в этих треугольниках. Условия согласования вместе с требованиями гладкости необходимы и достаточны для дважды непрерывной дифференцируемости решения на неявных характеристиках в этих прямоугольниках.

Ключевые слова: *общее телеграфное уравнение*, *неявные характеристики уравнения*, *критерий корректности*, *требование гладкости*, *условие согласования*.

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Abstract. The global theorem to Hadamard correctness to the first mixed problem for inhomogeneous general telegraph equation with all variable coefficients in a half-strip of the plane is proved by a novel method of auxiliary mixed problems. Without explicit continuations of the mixed problem data outside set of mixed task assignments the recurrent Riemann-type formulas of a unique and stable classical solution for the first mixed problem on a segment are derived. This half-strip of the plane is divided by the curvilinear characteristics of a telegraph equation into rectangles of the same height, and each rectangle into three triangles. The correctness criterion consists of smoothness requirements and matching conditions on the right-hand side of the equation, initial and boundary conditions of the mixed problem. The smoothness requirements are necessary and sufficient for twice continuous differentiability of the solution in these triangles. The matching conditions together with these smoothness requirements are necessary and sufficient for twice continuous differentiability of solution on the implicit characteristics in these rectangles.

Keywords: *general telegraph equation*, *implicit characteristics of equation*, *correctness criterion*, *smoothness requirement*, *matching condition.*

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Introduction

In this work the global correctness theorem (Theorem 2.1) to the first mixed problem for inhomoge-neous general telegraph equation with all variable coefficients in a half-strip of the plane is proved by a novel method of auxiliary mixed problems [1] from Theorem 1.1. Without explicit continuations of the problem data outside a set of mixed task assignments the recurrent Riemann-type formulas of a unique and stable classical (twice continuous differentiable) solution for the first mixed problem on a segment are derived. The correctness criterion for this mixed problem consists of smoothness requirements and six matching conditions to mixed problem data. Theorem 1.1 to the auxiliary first mixed problem for inhomogeneous general telegraph equation with all variable coefficients in the first quarter of the plane was established by a modification of the Riemann method. Note that for the first time a different type formula for a solution was obtained and the existence of a unique and stable classical solution of this auxiliary mixed problem was shown by Schauder's method of continuation with respect to parameter and the author's theorems on increasing the smoothness of strong generalized solutions in the work [2]. This article indicates necessary and sufficient smoothness requirements for the boundary and initial data, only sufficient smoothness requirements for the right-hand side of the equation and necessary and sufficient matching conditions for the boundary and initial data and the right-hand side of the equation. For this auxiliary mixed problem, the necessary and sufficient smoothness requirements on the right-hand side of this general telegraph equation are found using the correcting Goursat problem by the author's correction method, as for the model telegraph equation at variable rate $a(x,t)$ in [3].

For a concise and accurate assessment of the results of scientific work, we introduced the concept of global (and hence local) solvability theorems for linear boundary value and initial boundary (mixed) problems in works [briefly 4 and in detail 5]. The global correctness theorem of the first mixed problem for a one-dimensional wave equation with constant rate $a(x,t)=a=const>0$ in a half-strip of the plane has also been proven in [4], [5]. For the first time in this work, a critical analysis of the computation of explicit solutions of linear boundary value problems by modern methods is made. The possibility of deriving global theorems by special non-periodic continuations of the input data of these correctly posed boundary value problems is substantiated. Global theorems are understood as theorems with the weakest (necessary and sufficient) assumptions on the mixed problem data of these problems.

Definition [4], [5]. A solvability theorem of a boundary value problem in a pair of locally convex topological vector spaces is called *global* if its assumptions are necessary and sufficient conditions for the Hadamard correctness of this boundary value problem.

The global correctness theorem for a boundary value problem contains a criterion (necessary and sufficient conditions) for its correctness (according to Hadamard: existence, uniqueness and continuous dependence of a solution on a problem data). There is an infinite set of all possible extensions of the problem data for each boundary value problem. In fact, for each continuation method, we have our own solution, our own sufficient conditions for correctness and, thus, only a certain local correctness theorem of the boundary value problem. Local correctness theorems for the boundary value problem contain only sufficient conditions for their correctness. The non-continuation of the problem data of the boundary value problem serves as a sign of the globality of the derived theorem of their correctness. Nevertheless, using Zorn's lemma, we have proved a theorem on the possibility of deriving global correctness theorems by special extensions of problem data for linear boundary value problems.

Theorem [4], [5]. *Each well-posed linear boundary value problem for a partial differential equation has a global theorem of its correct Hadamard solvability in the corresponding pair of locally convex topological vector spaces.*

Our mixed problem (2.1)–(2.3) in theorem 2.1, due to the rate $a(x,t)$ dependent on *x* and *t*, does not admit the use of the Fourier method (separation of variables), the generalization of which is used to solve all mixed problems in [6]–[14]. In them, solutions of mixed problems for string vibration equation (2.1) with coefficients $a=1$, $b=c=0$ and a potential $q=q(x)$ are sought by the Khromov method, which is understood as a modification of the Fourier method by using the resolvent method, the ideas of A.N. Krylov on the acceleration of the convergence of Fourier series and L. Euler's ideas on divergent series. Here the obtained Fourier series express generalized (almost classical, continuously differentiable) solutions of mixed problems that satisfy the string vibration equations on a segment only almost everywhere. These generalized solutions are obviously not classical solutions and their uniqueness is not proved, but assumed. In this work only sufficient correctness conditions for the right-hand side of string vibration equation are shown.

1 An auxiliary first mixed problem for the general telegraph equation with variable coefficients on the half-line

We have proved a global [4] correctness theorem on $\dot{G}_\infty =]0,+\infty[\times]0,+\infty[$ to the problem:

$$\begin{aligned}\mathcal{L}u \equiv u_{tt}(x,t) - a^2(x,t)u_{xx}(x,t) + b(x,t)u_t(x,t) + \\ +c(x,t)u_x(x,t) + q(x,t)u(x,t) = \\ = f(x,t), (x,t) \in \dot{G}_\infty,\end{aligned} \quad (1.1)$$

$$u|_{t=0} = \varphi(x),\ u_t|_{t=0} = \psi(x),\ x>0, \quad (1.2)$$

$$u|_{x=0} = \mu(t),\ t>0, \quad (1.3)$$

where the subscripts of the function *u* denote its quotients derivatives of the corresponding orders with respect to the indices indicated variables, $a_1>0$, $a_2>0$ are real constants, coefficients and problem data f,φ,ψ,μ – given functions of their variables *x* and *t*.

Let $C^k(\Omega)$ be the set of k times continuously differentiable functions on the subset $\Omega\subset\mathbb{R}^2$ and $C^0(\Omega)=C(\Omega)$.

Definition **1.1.** *The classical solution* to the mixed problem (1.1)–(1.3) on $\dot{G}_\infty$ is called the function $u\in C^2(G_\infty)$, which satisfies equation (1.1) in the usual sense on $\dot{G}_\infty$, and the initial conditions (1.2) and boundary regime (1.3) in the sense of the limits corresponding expressions from its values $u(\dot{x},\dot{t})$ in interior points $(\dot{x},\dot{t})\in\dot{G}_\infty$ for $\dot{x}\to x$, $\dot{t}\to t$ for all indicated boundary points (x, t).

The characteristic equations
$$dx-(-1)^i a(x,t)dt=0$$
give the implicit characteristics $g_i(x,t)=C_i, i=1,2.$ If $a(x,t)\ge a_0>0,$ then they decrease strictly in t at $i=1$ and increase at $i=2$ with increasing x. Therefore, the functions $y_i=g_i(x,t)$ have inverse functions $x=h_i\{y_i,t\}, t=h^{(i)}[x,y_i].$ If $a\in C^2(G_\infty),$ then the functions $g_i,h_i,h^{(i)}\in C^2$ to the variables $x,t,y_i,$ $i=1,2$ [2].

By the definition of inverse mappings, they satisfy the following inversion identities from [2]:
$$g_i(h_i\{y_i,t\},t)=y_i,\forall y_i,\quad h_i\{g_i(x,t),t\}=x,\ x\ge 0,\ i=1,2, \tag{1.4}$$
$$g_i(x,h^{(i)}[x,y_i])=y_i,\forall y_i,\quad h^{(i)}[x,g_i(x,t)]=t,\ t\ge 0,\ i=1,2, \tag{1.5}$$
$$h_i\{y_i,h^{(i)}[x,y_i]\}=x,\ x\ge 0,\quad h^{(i)}[h_i\{y_i,t\},y_i]=t,\ t\ge 0,\ i=1,2. \tag{1.6}$$

The critical characteristic $g_2(x,t)=g_2(0,0)$ divides the first quarter plane $G_\infty=[0,+\infty[\times\ [0,+\infty[$ into two sets $G_-=\{(x,t)\in G_\infty: g_2(x,t)>g_2(0,0)\}$ and $G_+=\{(x,t)\in G_\infty: g_2(x,t)\le g_2(0,0)\}$. By a modification of the Riemann method it has been proved

***Theorem* 1.1** [2], [3], [15]. *Let the coefficients of the equation* (1.1) *be* $a(x,t)\ge a_0>0,(x,t)\in G_\infty,$ $a\in C^2(G_\infty),$ $b,c,q\in C^1(G_\infty).$ *The first mixed problem* (1.1)–(1.3) *in the set* $\dot{G}_\infty$ *has a unique and stable according to* φ,ψ,f,μ *classical solution* $u\in C^2(G_\infty), G_\infty=[0,+\infty[\times[0,+\infty[,$ *if and only if the following smoothness requirements and matching conditions are true* :
$$\varphi\in C^2[0,+\infty[,\ \psi\in C^1[0,+\infty[,\quad \mu\in C^2[0,+\infty[,\ f\in C(G_\infty), \tag{1.7}$$
$$\int_0^t f(|h_i\{g_i(x,t),\tau\}|,\tau)d\tau\in C^1(G_\infty), i=1,2, \tag{1.8}$$
$$\varphi(0)=\mu(0),\psi(0)=\mu'(0),\quad f(0,0)+a^2(0,0)\varphi''(0)-b(0,0)\psi(0)-c(0,0)\varphi'(0)-q(0,0)\varphi(0)=\mu''(0). \tag{1.9}$$
The classical solution $u\in C^2(G_\infty)$ *to the first mixed problem* (1.1)–(1.3) *in* $\dot{G}_\infty$ *is the function*
$$u_-(x,t)=\frac{1}{2a(x,t)}\big((auv)(h_2\{g_2(x,t),0\},0)+(auv)(h_1\{g_1(x,t),0\},0)\big)+\frac{1}{2a(x,t)}\int_{h_2\{g_2(x,t),0\}}^{h_1\{g_1(x,t),0\}}[\psi(s)v(s,0)-\varphi(s)v_\tau(s,0)+b(s,0)\varphi(s)v(s,0)]ds+\frac{1}{2a(x,t)}\int_0^t d\tau\int_{h_2\{g_2(x,t),\tau\}}^{h_1\{g_1(x,t),\tau\}} f(s,\tau)v(s,\tau)ds,(x,t)\in G_-, \tag{1.10}$$
$$u_+(x,t)=\frac{1}{2a(x,t)}\big((auv)(h_1\{g_1(x,t),0\},0)-(auv)(h_1\{g_1(0,h^{(2)}[0,g_2(x,t)]),0\},0)\big)+\frac{1}{2a(x,t)}\int_{h_1\{g_1(0,h^{(2)}[0,g_2(x,t)]),0\}}^{h_1\{g_1(x,t),0\}}[\psi(s)v(s,0)-\varphi(s)v_\tau(s,0)+b(s,0)\varphi(s)v(s,0)]ds+\frac{1}{2a(x,t)}\int_0^t d\tau\int_{h_2\{g_2(x,t),\tau\}}^{h_1\{g_1(x,t),\tau\}}\breve{f}(|s|,\tau)v(|s|,\tau)ds+\mu(t)-\frac{1}{2a(0,t)}\int_0^t d\tau\int_{h_2\{g_2(0,t),\tau\}}^{h_1\{g_1(0,t),\tau\}}\breve{f}(|s|,\tau)v(|s|,\tau)ds,(x,t)\in G_+, \tag{1.11}$$
where the functions are
$$\breve{f}(x,t)=f(x,t)+f^{(0)}(x,t)-f_\mu(x,t),$$
$f_\mu(x,t)=\mathcal{L}\mu(t),$ *and* $f^{(0)}$ *is the restriction on the set* G_∞ *of the solution to the corresponding system of the Volterra integral equation of the second kind and the linear algebraic equation.*

In G_- the Riemann function $v(s,\tau)=v(s,\tau;x,t)$ is the solution to the Goursat problem:
$$v_{\tau\tau}(s,\tau)-(a^2(s,\tau)v(s,\tau))_{ss}-(b(s,\tau)v(s,\tau))_\tau-(c(s,\tau)v(s,\tau))_s+q(s,\tau)v(s,\tau)=0,\quad (s,\tau)\in\Delta MPQ, \tag{1.12}$$
$$v(s,\tau)=\exp\left\{\int_t^\tau k_1(h_1\{g_1(x,t),\rho\},\rho)d\rho\right\},\quad g_1(s,\tau)=g_1(x,t),$$
$$v(s,\tau)=\exp\left\{\int_t^\tau k_2(h_2\{g_2(x,t),\rho\},\rho)d\rho\right\},\quad g_2(s,\tau)=g_2(x,t),\ \tau\in[0,t], \tag{1.13}$$
where the functions
$$k_1(s,\tau)=\{a(s,\tau)b(s,\tau)+3a(s,\tau)a_s(s,\tau)-a_\tau(s,\tau)+c(s,\tau)\}/4a(s,\tau)$$
on the curve QM and
$$k_2(s,\tau)=\{a(s,\tau)b(s,\tau)-3a(s,\tau)a_s(s,\tau)-a_\tau(s,\tau)-c(s,\tau)\}/4a(s,\tau)$$
on the curve MP of the curvilinear characteristic triangle ΔMPQ (see Figure 1.1).

In G_+ the Riemann function $\check{v}(s,\tau)=\check{v}(s,\tau;x,t)$ is the solution to the Goursat problem:
$$\check{v}_{\tau\tau}(s,\tau)-(\check{a}^2(s,\tau)\check{v}(s,\tau))_{ss}-(\breve{b}(s,\tau)v(\check{s},\tau))_\tau-(\tilde{c}(s,\tau)\check{v}(s,\tau))_s+\check{q}(s,\tau)\check{v}(s,\tau)=0,\quad (s,\tau)\in\Delta MPQ, \tag{1.14}$$

$$\breve{v}(s,\tau)=\exp\left\{\int_t^\tau \breve{k}_1(h_1\{g_1(x,t),\rho\},\rho)d\rho\right\},$$
$$g_1(s,\tau)=g_1(x,t),$$
$$\breve{v}(s,\tau)=\exp\left\{\int_t^\tau \breve{k}_2(h_2\{g_2(x,t),\rho\},\rho)d\rho\right\}, \quad (1.15)$$
$$g_2(s,\tau)=g_2(x,t),\ \tau\in[0,t],$$

with functions $\breve{k}_1(s,\tau)$ on the curve QM and $\breve{k}_2(s,\tau)$ on the curve MP, respectively equal to the functions $k_1(s,\tau)$ and $k_2(s,\tau)$, in which the coefficients *a*, *b*, *c*, *q* are respectively replaced by their even in *x* extensions $\breve{a},\breve{b},\breve{q}$ and odd in *x* extension $\tilde{c}$ of the coefficients *a*, *b*, *c*, *q* (see Figure 1.2).

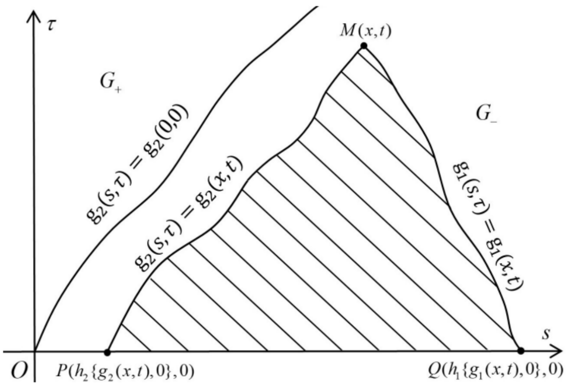

Figure 1.1 – Curvilinear characteristic triangle $\vartriangle MPQ$ for the vertex $M\in G_-$.

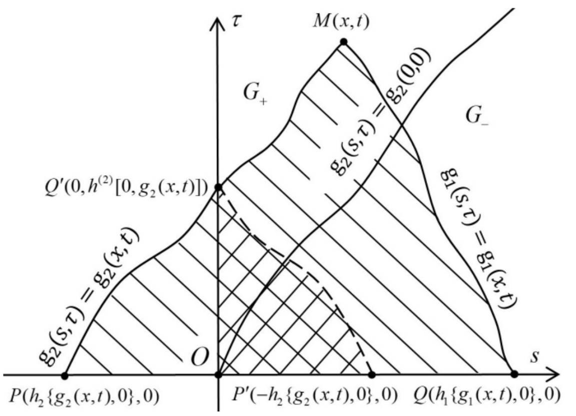

Figure 1.2 – Curvilinear characteristic and critical triangles $\vartriangle MPQ$ and $\vartriangle Q'PP'$ respectively for the vertex $M\in G_+$.

At each fixed point $M(x,t)\in G_\infty$ the tangent of the inclination angles of tangent lines to the curvilinear characteristics $g_i(x,t)=C_i, i=1,2,$ differ only in opposite signs $dx/dt=(-1)^i a(x,t), i=1,2,$ (see Figure 1.1, Figure 1.2). But since the extensions $\tilde{a}$, $\tilde{b}$, $\tilde{q}$, $\tilde{f}$ are even and $\tilde{c}$ is odd along *s* relative to the axis $O\tau$ for any vertex $M(0,t), t>0,$ lying on the axis $O\tau$, curvilinear characteristic triangles $\vartriangle MPQ$ and, in the particular, the triangles $\vartriangle Q'PP'$ are "isosceles" (see Figure 1.2).

It is proved that the Goursat problems (1.12), (1.13) and (1.14), (1.15) with coefficients $a\in C^2(G_\infty)$, $b,c,q\in C^1(G_\infty)$ always have the only classical solutions $v\in C^2$ on G_- and G_+. The formula (1.11) of the classical solution u_+ to this problem on G_+ does not contain the values of the extensions $\breve{a},\breve{b}$, $\tilde{c},\breve{q},\breve{f},\breve{f}_\mu,\breve{f}^{(0)}$ for $x<0$, as in formula (1.10), therefore that these extensions turned out to be formal due to the modulus sign $|s|$ in the functions $\breve{f}(|s|,\tau)$ и $v(|s|,\tau)$. Therefore, in the solution (1.11), the first iterated integral, which is equal to the double integral over the characteristic triangle ΔMPQ, is actually taken over the curvilinear quadrangle $MQ'P'Q$ and twice over the triangle $\vartriangle Q'OP'$ of the critical triangle $\vartriangle Q'PP'$ of the product $\breve{f}(|s|,\tau)v(|s|,\tau)$.

***Corollary* 1.1.** *If the continuous right-hand side* $f\in C[0,+\infty[$ *depends only on x or t, then the assertion of this Theorem* 1.1 *is true without integral smoothness requirements* (1.8).

For a function *f* depending only on *x* or *t* and continuous in Q_n, the integral requirements (1.8) in Theorem 1.1 are automatically satisfied.

***Corollary* 1.2.** *Let the coefficients of the equation* (1.1) *be* $a(x,t)\ge a_0>0,(x,t)\in G_\infty$, $a\in C^2(G_\infty)$, $b,c,q\in C^1(G_\infty)$. *If the right-hand side f depends on x and t, then in the smoothness requirements* (1.8) *on* G_∞ *the belonging of integrals to a set* $C^1(G_\infty)$ *are equivalent to their belonging to sets* $C^{(0,1)}(G_\infty)$ *or* $C^{(1,0)}(G_\infty)$. *Here* $C^{(0,1)}(\Omega)$ $(C^{(0,1)}(\Omega))$ *is the set of all continuous* (*continuously differentiable*) *with respect to x and continuously differentiable* (*continuous*) *with respect to t functions on* Ω.

The proof of Corollary 1.2 is similar to its proof in the case of constant coefficients $a(x,t)=const>0,\ b=c=q=0$ of the equation (1.1) in chapter 2 of from the candidate dissertation [15].

***Remark* 1.1.** It was first proved the existence of a unique and stable classical solution to the mixed problem (1.1)–(1.3) by the Schauder's continuation method with respect to a parameter and the author's theorems on increasing the smoothness of strong solutions in the article [2].

2 The main first mixed problem for the general telegraph equation with variable coefficients on a segment

We need to decide and derive the correctness criterion on $\dot{Q}_n=]0,d[\times]0,d_{n+1}[$ of the problem:

$$\mathcal{L}u \equiv u_{tt}(x,t) - a^2(x,t)u_{xx}(x,t) + b(x,t)u_t(x,t) + \\ + c(x,t)u_x(x,t) + q(x,t)u(x,t) = \\ = f(x,t), (x,t) \in \dot{Q}_n, \quad (2.1)$$

$$u|_{t=0} = \varphi(x), u_t|_{t=0} = \psi(x), 0 < x < d, \quad (2.2)$$

$$u|_{x=0} = \mu_1(t), u|_{x=d} = \mu_2(t), 0 < t < d_{n+1}, \\ d_n = (n-1)h^{(2)}[d/2, g_2(0.0)]. \quad (2.3)$$

Let us find in an explicit form the classical solutions of this mixed problem and the criterion for its Hadamard's correctness.

***Definition* 2.1.** *The classical solution* to the mixed problem (2.1)–(2.3) on $\dot{Q}_n$ is called the function $u \in C^2(Q_n)$, $Q_n = [0,d]\times[0,d_{n+1}]$, which satisfies equation (2.1) in the usual sense on $\dot{Q}_n$, and the initial conditions (2.2) and boundary regimes (2.3) in the sense of the limits corresponding expressions from its values $u(\dot{x},\dot{t})$ in interior points $(\dot{x},\dot{t}) \in \dot{Q}_n$ for $\dot{x} \to x$, $\dot{t} \to t$ for all indicated boundary points (x, t).

The statement of the mixed problem (2.1)–(2.3) and the definition 2.1 of its classical solutions $u \in C^2(Q_n)$ imply the obvious necessary smoothness requirements

$$f \in C(Q_n), \varphi \in C^2[0,d], \\ \psi \in C^1[0,d], \mu_1, \mu_2 \in C^2[0,d_{n+1}]. \quad (2.4)$$

To obtain the first four matching conditions the boundary regimes (2.3) with initial conditions (2.2) and equation (2.1) in equalities (2.3) and the first derivative with respect to t of these equalities, we set $t = 0$ and use the initial data

$$\varphi(\hat{d}_p) = \mu_p(0), \psi(\hat{d}_p) = \mu_{p'}(0), \\ \hat{d}_p = (p-1)d, p = 1,2. \quad (2.5)$$

To find two more matching conditions, we differentiate equalities (2.3) twice in t, we calculate the values of the derivatives of solutions u for $x = 0, t = 0$ and $x = d, t = 0$ using initial conditions (2.2) and equation (2.1) and we obtain

$$f(\hat{d}_p,0) + a^2(\hat{d}_p,0)\varphi''(\hat{d}_p) - b(\hat{d}_p,0)\psi(\hat{d}_p) - \\ -c(\hat{d}_p,0)\varphi'(\hat{d}_p) - q(\hat{d}_p,0)\varphi(\hat{d}_p) = \mu_{p''}(0), \quad (2.6) \\ p = 1,2.$$

We denote by the number of strokes over functions with one variable the corresponding orders of their ordinary derivatives with respect to these variables.

A global correctness Theorem 2.1 to this first mixed problem (2.1)–(2.3) is derived from Theorem 1.1 "by the method of auxiliary mixed problems for a semi-bounded string (wave equation on a half-line)" from [1]. In the limit at $n \to +\infty$, bounded rectangles Q_n exhaust the half-strip $G = [0,d]\times[0,+\infty[$, unbounded along the time variable t. For global correctness Theorem 2.1, the half-strip G is divided into rectangles $G_n = [0,d]\times[d_n, d_{n+1}]$, where $d_n = (n-1)h^{(2)}[d/2, g_2(0,0)]$, $n = 1,2,3,...$, each of which is divided by the critical characteristics $g_2(x,t) = g_2(0,d_n)$, $g_1(x,t) = g_1(d,d_n)$, $n = 1,2,3,...$, into triangles:

$$\Delta_{3n-2} = \{(x,t) \in G : g_2(x,t) \ge g_2(0,d_n), \\ g_1(x,t) \le g_1(d,d_n), x \in [0,d], t \in [d_n, d_{n+1}]\},$$

$$\Delta_{3n-1} = \{(x,t) \in G : g_2(x,t) \le g_2(0,d_n), \\ x \in [0,d/2], t \in [d_n, d_{n+1}]\},$$

$$\Delta_{3n} = \{(x,t) \in G : g_1(x,t) \ge g_1(d,d_n), \\ x \in [d/2,d], t \in [d_n, d_{n+1}]\}, n = 1,2,3,...$$

We have proved by a novel method of auxiliary mixed problems the following global correctness

***Theorem* 2.1.** *Let the coefficients of the equation* (2.1) *be* $a(x,t) \ge a_0 > 0, (x,t) \in Q_n$, $a \in C^2(Q_n)$, $b,c,q \in C^1(Q_n)$. *The first mixed problem* (2.1)–(2.3) *in the set* $\dot{Q}_n$ *has a unique and stable according to* $\varphi, \psi, f, \mu_1, \mu_2$ *classical solution* $u \in C^2(Q_n)$, $Q_n = [0,d]\times[0,d_{n+1}]$, *if and only if the following smoothness requirements are true* (2.4),

$$\int_{d_k}^{t} f\left(|h_i\{g_i(x,t),\tau\}|,\tau\right)d\tau \in C^1(\Delta_{3k-2} \cup \Delta_{3k-1}), \quad (2.7) \\ i = 1,2,$$

$$\int_{d_k}^{t} f\left(d - |d - h_i\{g_i(x,t),\tau\}|,\tau\right)d\tau \in C^1(\Delta_{3k-2} \cup \Delta_{3k}), \quad (2.8) \\ i = 1,2,$$

for all indices $k = \overline{1,n}, n = 1,2,...,$ *and the matching conditions* (2.5), (2.6).

The classical solution to the first mixed problem (2.1)–(2.3) in rectangle $\dot{Q}_n$ is the function

$$u_{3k-2}(x,t) = \frac{1}{2a(x,t)}\Big((auv)(h_2\{g_2(x,t),d_k\},d_k) + \\ +(auv)(h_1\{g_1(x,t),d_k\},d_k)\Big) + \\ +\frac{1}{2a(x,t)}\int_{h_2\{g_2(x,t),d_k\}}^{h_1\{g_1(x,t),d_k\}}[\psi_k(s)v(s,d_k) - \varphi_k(s)v_\tau(s,d_k) + \\ +b(s,d_k)\varphi_k(s)v(s,d_k)]ds + \quad (2.9) \\ +\frac{1}{2a(x,t)}\int_{d_k}^{t}d\tau\int_{h_2\{g_2(x,t),\tau\}}^{h_1\{g_1(x,t),\tau\}} f(s,\tau)v(s,\tau)ds, (x,t) \in \Delta_{3k-2},$$

$$u_{3k-1}(x,t) = \frac{1}{2a(x,t)}\Big((auv)(h_1\{g_1(x,t),d_k\},d_k) - \\ -(auv)(h_1\{g_1(0,h^{(2)}[0,g_2(x,t)]),d_k\},d_k)\Big) + \\ +\frac{1}{2a(x,t)}\int_{h_1\{g_1(0,h^{(2)}[0,g_2(x,t)]),d_k\}}^{h_1\{g_1(x,t),d_k\}}[\psi_k(s)v(s,d_k) - \quad (2.10) \\ -\varphi_k(s)v_\tau(s,d_k) + b(s,d_k)\varphi_k(s)v(s,d_k)]ds + \\ +\frac{1}{2a(x,t)}\int_{d_k}^{t}d\tau\int_{h_2\{g_2(x,t),\tau\}}^{h_1\{g_1(x,t),\tau\}}\breve{f}_1(|s|,\tau)v(|s|,\tau)ds + \mu_1(t) - \\ -\frac{1}{2a(0,t)}\int_{d_k}^{t}d\tau\int_{h_2\{g_2(0,t),\tau\}}^{h_1\{g_1(0,t),\tau\}}\breve{f}_1(|s|,\tau)v(|s|,\tau)ds, (x,t) \in \Delta_{3k-1},$$

$$
\begin{gathered}
u_{3k}(x,t)=\frac{1}{2a(x,t)}\Big((auv)(h_2\{g_2(x,t),d_k\},d_k)-\\
-(auv)(h_2\{g_2(d,h^{(1)}[d,g_1(x,t)]),d_k\},d_k)\Big)+\\
+\frac{1}{2a(x,t)}\int\limits_{h_2\{g_2(x,t),d_k\}}^{h_2\{g_2(d,h^{(1)}[d,g_1(x,t)]),d_k\}}[\psi_k(s)v(s,d_k)-\\
-\varphi_k(s)v_\tau(s,d_k)+b(s,d_k)\varphi_k(s)v(s,d_k)]ds+\\
+\frac{1}{2a(x,t)}\int\limits_{d_k}^{t}d\tau\int\limits_{d-h_1\{g_1(x,t),\tau\}}^{d-h_2\{g_2(x,t),\tau\}}\breve{f}_2(d-|s|,\tau)v(d-|s|,\tau)ds+\\
+\mu_2(t)-\\
-\frac{1}{2a(0,t)}\int\limits_{d_k}^{t}d\tau\int\limits_{d-h_1\{g_1(0,t),\tau\}}^{d-h_2\{g_2(0,t),\tau\}}\breve{f}_2(d-|s|,\tau)v(d-|s|,\tau)ds,\\
(x,t)\in\Delta_{3k},
\end{gathered}
\tag{2.11}
$$

for all indices $k=\overline{1,n},\ n=1,2,\ldots$ *Here the functions* u_{3k-l} *are the restrictions of the solution* u *to the problem* (2.1)–(2.3) *on triangles* $\Delta_{3k-l},\ l=0,1,2,$ *and recurrent initial data are equal*

$$
\begin{gathered}
\varphi_1(x)=\varphi(x),\ \psi_1(x)=\psi(x),\\
x\in[0,d],\ \varphi_k(x)=u_{3k+j-4}\Big|_{t=d_k},
\end{gathered}
\tag{2.12}
$$

$$
\psi_k(x)=\partial_t\,u_{3k+j-4}\Big|_{t=d_k},\ x\in[j(d/2),(j+1)(d/2)],
$$

$$
j=0,1,\ k=\overline{2,n},\ n=1,2,\ldots
$$

The functions

$$
\breve{f}_p(x,t)=f(x,t)+f_p^{(0)}(x,t)-f_{\mu_p}(x,t),
$$

$$
f_{\mu_p}(x,t)=\mathcal{L}\mu_p(t),
$$

$f_p^{(0)}(x,t)$ *and Riemann functions* $v(x,t)$ *are the restrictions on triangles* Δ_{3k-l} *of the same functions from Theorem* 1.1.

Proof. Theorem 2.1 will be proved by the method of mathematical induction over rectangles Q_n. At the first step of mathematical induction for the mixed problem (2.1)–(2.3) on a rectangle $Q_1=G_1$ we will verify the existence of a unique and stable classical solution $u\in C^2(Q_1)$ of the form (2.9)–(2.11) with correctness criterion (2.4)–(2.8) for $k=n=1$. The restrictions of necessary and sufficient conditions (1.7)–(1.9) and formulas (1.10), (1.11) at $\varphi(x)=\varphi_1(x),\quad \psi(x)=\psi_1(x), x\in[0,d],$ $\mu(t)=\mu_1(t),\ \ t\in[0,d_2],$ from Theorem 1.1 onto the trapezoid $\Delta_1\bigcup\Delta_2$, respectively, coincide with the correctness criterion (2.4)–(2.8) for $k=n=1,\ \ p=1$ and formulas (2,9), (2.10) from Theorem 2.1. In the trapezoid $\Delta_1\bigcup\Delta_2$, the correctness criterion consists of the smoothness requirements (2.4), (2.7) for $k=n=1$ and matching conditions (2.5), (2.6) at $p=1$.

To find the classical solution and the correctness criterion of the mixed problem consisting of the equation (2.1), initial conditions (2.2), and the second boundary regime from (2.3) at $x=d$ in the trapezoid $\Delta_1\bigcup\Delta_3$, we reduce it by replacing $x=d-\tilde{x},\ \ t=\tilde{t}$ to the equivalent mixed problem

$$
\begin{gathered}
\tilde{u}_{tt}(\tilde{x},t)-\tilde{a}^2(\tilde{x},t)u_{\tilde{x}\tilde{x}}(\tilde{x},t)+\tilde{b}(\tilde{x},t)\tilde{u}_t(\tilde{x},t)+\\
+\tilde{c}(\tilde{x},t)\tilde{u}_{\tilde{x}}(\tilde{x},t)+\tilde{q}(\tilde{x},t)\tilde{u}(\tilde{x},t)=\\
=\tilde{f}(\tilde{x},t),(\tilde{x},t)\in\tilde{\Delta}_1\bigcup\tilde{\Delta}_2,
\end{gathered}
\tag{2.13}
$$

$$
\tilde{u}\,|_{t=0}=\tilde{\varphi}(\tilde{x}),\ \tilde{u}_t\,|_{t=0}=\tilde{\psi}(\tilde{x}),\ \tilde{x}\in[0,d],\tag{2.14}
$$

$$
\tilde{u}\,|_{\tilde{x}=0}=\mu_2(t),\ t\in[0,d_2],\tag{2.15}
$$

relatively new function $\tilde{u}(\tilde{x},t)=u(d-\tilde{x},t)=u(x,t)$ with new coefficients $\tilde{a}(\tilde{x},t)=a(d-\tilde{x},t)=a(x,t),$ $\tilde{b}(\tilde{x},t)=b(d-\tilde{x},t)=b(x,t),\qquad \tilde{c}(\tilde{x},t)=c(d-\tilde{x},t)=$ $=c(x,t),\ \ \tilde{q}(\tilde{x},t)=q(d-\tilde{x},t)=q(x,t)$ and problem data $\tilde{f}(\tilde{x},t)=f(d-\tilde{x},t)=f(x,t),\ \ \tilde{\varphi}(\tilde{x})=\varphi(d-\tilde{x})=$ $=\varphi(x),\ \ \tilde{\psi}(\tilde{x})=\psi(d-\tilde{x})=\psi(x).$ Here the trapezoid $\tilde{\Delta}_1\bigcup\tilde{\Delta}_2$ is formed by two triangles

$$
\begin{gathered}
\tilde{\Delta}_1=\{(x,t)\in G_\infty: g_1(x,t)\geq g_1(0,0),\\
g_2(x,t)\leq g_2(0,0),\ x\in[0,d],\ t\in[0,d_2]\},\\
\tilde{\Delta}_2=\{(x,t)\in G_\infty: g_1(x,t)\leq g_1(0,0),\ x\in[0,d/2],\\
t\in[0,d_2]\},\ d_n=(n-1)h^{(1)}[d/2,g_1(d,0)].
\end{gathered}
$$

After this non-degenerate replacement $x=d-\tilde{x},$ $t=\tilde{t}$ the implicit characteristic functions $y_i=g_i(x,t)=C_i,\ x,t\geq 0,$ and their inverse functions $x=h_i\{y_i,t\},\ t\geq 0,\ \ t=h^{(i)}[x,y_i],\ x\geq 0,\ i=1,2,$ become the functions:

$$
\tilde{y}_i=\tilde{g}_i(\tilde{x},t)=g_i(d-\tilde{x},t)=g_i(x,t),\ x,t\geq 0,\tag{2.16}
$$

$$
\tilde{x}=\tilde{h}_i\{\tilde{y}_i,t\}=d-h_i\{\tilde{y}_i,t\},\ t\geq 0,\tag{2.17}
$$

$$
\begin{gathered}
t=\tilde{h}^{(i)}[\tilde{x},\tilde{y}_i]=h^{(i)}[d-\tilde{x},\tilde{y}_i]=h^{(i)}[x,\tilde{y}_i],\\
x\geq 0,\ i=1,2.
\end{gathered}
\tag{2.18}
$$

For them, analogous inversion identities are derived from the inversion identities (1.4)–(1.6) respectively:

$$
\begin{gathered}
\tilde{g}_i(\tilde{h}_i\{\tilde{y}_i,t\},t)=\tilde{y}_i,\ \forall\tilde{y}_i,\ \tilde{h}_i\{\tilde{g}_i(\tilde{x},t),t\}=\tilde{x},\\
\tilde{x}\geq 0,\ i=1,2,\\
\tilde{g}_i(\tilde{x},\tilde{h}^{(i)}[\tilde{x},\tilde{y}_i])=\tilde{y}_i,\ \forall\tilde{y}_i,\ \tilde{h}^{(i)}[\tilde{x},\tilde{g}_i(\tilde{x},t)]=t,\\
t\geq 0,\ i=1,2,\\
\tilde{h}_i\{\tilde{y}_i,\tilde{h}^{(i)}[\tilde{x},\tilde{y}_i]\}=\tilde{x},\ \tilde{x}\geq 0,\ \tilde{h}^{(i)}[\tilde{h}_i\{\tilde{y}_i,t\},\tilde{y}_i]=t,\\
t\geq 0,\ i=1,2.
\end{gathered}
$$

According to Theorem 1.1, the unique and stable classical solution $\tilde{u}(\tilde{x},t)$ to the mixed problem (2.13)–(2.15) in a triangle $\tilde{\Delta}_1$ is given by the restriction of the unique and stable classical solution $\tilde{u}_-(\tilde{x},t)$ to $\tilde{\Delta}_1$ the form (1.10), in which the characteristic functions $\tilde{g}_1,\tilde{h}_1,\tilde{h}^{(1)}$ are replaced respectively by functions $\tilde{g}_2,\tilde{h}_2,\tilde{h}^{(2)}$ and vice versa, the coefficients a, b, c, q – by the coefficients $\tilde{a},\tilde{b},-\tilde{c},\tilde{q}$ and the lateral sides MP, QM – by the lateral sides $\tilde{Q}\tilde{M}$, $\tilde{M}\tilde{P}$ of the characteristic triangles ΔMPQ and $\Delta\tilde{M}\tilde{P}\tilde{Q}$. The vertices of these characteristic triangles are points $\tilde{M}(\tilde{x},t)=M(d-\tilde{x},t)=M(x,t)$,

$$\tilde{P}(\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),0\},0)=P(d-\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),0\},0)=$$
$$P(h_2\{g_2(d-\tilde{x},t),0\},0)=P(h_2\{g_2(x,t),0\},\ 0),$$
$$\tilde{Q}(\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),0\},0)=Q(d-\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),0\},0)=$$
$$Q(h_1\{g_1(d-\tilde{x},t),0\},0)=Q(h_1\{g_1(x,t),0\},0).$$

The last changes of the sides of the triangles ΔMPQ and $\Delta\tilde{M}\tilde{P}\tilde{Q}$ in the formula (1.10) mean the replacements of the functions k_1 and k_2, respectively, by the functions k_2 and k_1 in the Goursat problem (1.12), (1.13).

As a result, from formula (1.10) at $\varphi(x)=\varphi_1(x)$, $\psi(x)=\psi_1(x), x\in[0,d]$, for $(\tilde{x},t)\in\tilde{\Delta}_1$ we have the unique and stable classical solution

$$\tilde{u}_1(\tilde{x},t)=\frac{1}{2\tilde{a}(\tilde{x},t)}\Big((\tilde{a}\tilde{u}\tilde{v})(\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),0\},0)+$$
$$+(\tilde{a}\tilde{u}\tilde{v})(\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),0\},0)\Big)+$$
$$+\frac{1}{2\tilde{a}(\tilde{x},t)}\int\limits_{\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),0\}}^{\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),0\}}[\tilde{\psi}_1(s)\tilde{v}(s,0)-\tilde{\varphi}_1(s)\tilde{v}_\tau(s,0)+$$
$$+\tilde{b}(s,0)\tilde{\varphi}_1(s)\tilde{v}(s,0)]ds+ \tag{2.19}$$
$$+\frac{1}{2\tilde{a}(\tilde{x},t)}\int\limits_0^t d\tau\int\limits_{\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),\tau\}}^{\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),\tau\}}\tilde{f}(s,\tau)\tilde{v}(s,\tau)ds,(\tilde{x},t)\in\tilde{\Delta}_1.$$

Here the Riemann function $\tilde{v}(\tilde{s},\tau)=\tilde{v}(\tilde{s},\tau;\tilde{x},t)$ is classical solution on $\tilde{\Delta}_1$ to the Goursat problem:

$$\begin{gathered}\tilde{v}_{\tau\tau}(\tilde{s},\tau)-(\tilde{a}^2(\tilde{s},\tau)\tilde{v}(\tilde{s},\tau))_{\tilde{s}\tilde{s}}-(\tilde{b}(\tilde{s},\tau)\tilde{v}(\tilde{s},\tau))_\tau+\\+(\tilde{c}(\tilde{s},\tau)\tilde{v}(\tilde{s},\tau))_{\tilde{s}}+\tilde{q}(\tilde{s},\tau)\tilde{v}(\tilde{s},\tau)=0,\\(\tilde{s},\tau)\in\Delta\tilde{M}\tilde{P}\tilde{Q},\end{gathered} \tag{2.20}$$

$$\tilde{v}(\tilde{s},\tau)=\exp\left\{\int\limits_t^\tau\tilde{k}_2(\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),\rho\},\rho)d\rho\right\},$$
$$\tilde{g}_1(\tilde{s},\tau)=\tilde{g}_1(\tilde{x},t),$$
$$\tilde{v}(\tilde{s},\tau)=\exp\left\{\int\limits_t^\tau\tilde{k}_1(\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),\rho\},\rho)d\rho\right\}, \tag{2.21}$$
$$\tilde{g}_2(\tilde{s},\tau)=\tilde{g}_2(\tilde{x},t),\ \tau\in[0,t],$$

where the functions are

$$\tilde{k}_1(\tilde{s},\tau)=\{\tilde{a}(\tilde{s},\tau)\tilde{b}(\tilde{s},\tau)+3\tilde{a}(\tilde{s},\tau)\tilde{a}_{\tilde{s}}(\tilde{s},\tau)-$$
$$-\tilde{a}_\tau(\tilde{s},\tau)-\tilde{c}(\tilde{s},\tau)\}/4\tilde{a}(\tilde{s},\tau)$$

on the curve $\tilde{M}\tilde{P}$ and

$$\tilde{k}_2(\tilde{s},\tau)=\{\tilde{a}(\tilde{s},\tau)\tilde{b}(\tilde{s},\tau)-3\tilde{a}(\tilde{s},\tau)\tilde{a}_{\tilde{s}}(\tilde{s},\tau)-$$
$$-\tilde{a}_\tau(\tilde{s},\tau)+\tilde{c}(\tilde{s},\tau)\}/4\tilde{a}(\tilde{s},\tau)$$

on the curve $\tilde{Q}\tilde{M}$.

Since the derivative is $\tilde{a}_{\tilde{s}}(\tilde{s},\tau)=-a_s(s,\tau)$, then after the inverse replacement $\tilde{x}=d-x$ from the Goursat problem (2.20), (2.21) we arrive at the Goursat problem:

$$\begin{gathered}v_{\tau\tau}(s,\tau)-(a^2(s,\tau)v(s,\tau))_{ss}-(b(s,\tau)v(s,\tau))_\tau-\\-(c(s,\tau)v(s,\tau))_s+q(s,\tau)v(s,\tau)=0,\\(s,\tau)\in\Delta MPQ,\end{gathered} \tag{2.22}$$

$$v(s,\tau)=\exp\left\{\int\limits_t^\tau k_1(h_1\{g_1(x,t),\rho\},\rho)d\rho\right\},$$
$$g_1(s,\tau)=g_1(x,t),$$
$$v(s,\tau)=\exp\left\{\int\limits_t^\tau k_2(h_2\{g_2(x,t),\rho\},\rho)d\rho\right\}, \tag{2.23}$$
$$g_2(s,\tau)=g_2(x,t),\ \tau\in[0,t],$$

because

$$\tilde{k}_1(\tilde{s},\tau)=\{a(s,\tau)b(s,\tau)-3a(s,\tau)a_s(s,\tau)-$$
$$-a_\tau(s,\tau)-c(s,\tau)\}/4a(s,\tau)=k_2(s,\tau)$$

on the curve MP and

$$\tilde{k}_2(\tilde{s},\tau)=\{a(s,\tau)b(s,\tau)+3a(s,\tau)a_s(s,\tau)-$$
$$-a_\tau(s,\tau)+c(s,\tau)\}/4a(s,\tau)=k_1(s,\tau)$$

on the curve QM in Goursat conditions (2.21). Here we have used relations (2.16), (2.17) and

$$\tilde{k}_i(\tilde{s},\tau)=k_i(d-\tilde{s},\tau)=k_i(s,\tau),$$
$$\tilde{h}_i\{\tilde{g}_i(\tilde{s},\tau),\rho\}=d-h_i\{g_i(s,\tau),\rho\},$$
$$\begin{gathered}\tilde{k}_i\left(\tilde{h}_i\{\tilde{g}_i(\tilde{x},t),\rho\},\rho\right)=k_i\left(d-\tilde{h}_i\{\tilde{g}_i(\tilde{x},t),\rho\},\rho\right)=\\=k_i\left(h_i\{\tilde{g}_i(\tilde{x},t),\rho\},\rho\right)=k_i\left(h_i\{g_i(x,t),\rho\},\rho\right),\\i=1,2.\end{gathered} \tag{2.24}$$

So, in the formula (2.19) we make a change of variable $\tilde{x}=d-x$ using the following equalities:

$$\begin{gathered}(\tilde{a}\tilde{u}\tilde{v})(\tilde{h}_i\{\tilde{g}_i(\tilde{x},t),0\},0)=\\=(auv)(d-\tilde{h}_i\{\tilde{g}_i(\tilde{x},t),0\},0)=\\=(auv)(h_i\{\tilde{g}_i(\tilde{x},t),0\},0)=\\=(auv)(h_i\{g_i(d-\tilde{x},t),0\},0)=\\=(auv)(h_i\{g_i(x,t),0\},0),\\\tilde{h}_i\{\tilde{g}_i(\tilde{x},t),\tau\}=d-h_i\{\tilde{g}_i(\tilde{x},t),\tau\}=\\=d-h_i\{g_i(d-\tilde{x},t),\tau\}=\\=d-h_i\{g_i(d-x,t),\tau\},\ i=1,2,\end{gathered} \tag{2.25}$$

due to equalities (2.16), (2.17) and $\tilde{v}(\tilde{s},\tau)==v(d-\tilde{s},\tau)=v(s,\tau)$. By these substitution $\tilde{x}=d-x$ and transformations (2.25) from the classical solution (2.19) we find the classical solution

$$\tilde{u}_1(\tilde{x},t)=\frac{1}{2a(x,t)}\Big((auv)(h_1\{g_1(x,t),0\},0)+$$
$$+(auv)(h_2\{g_2(x,t),0\},0)\Big)+$$
$$+\frac{1}{2a(x,t)}\int\limits_{d-h_1\{g_1(x,t),0\}}^{d-h_2\{g_2(x,t),0\}}[\psi_1(d-s)v(d-s,0)-$$
$$-\varphi_1(d-s)v_\tau(d-s,0)+ \tag{2.26}$$
$$+b(d-s,0)\varphi_1(d-s)v(d-s,0)]ds+$$

$$+\frac{1}{2a(x,t)}\int_0^t d\tau \int_{d-h_1\{g_1(x,t),\tau\}}^{d-h_2\{g_2(x,t),\tau\}} f(d-s,\tau)v(d-s,\tau)ds =$$
$$= u_1(x,t), (x,t)\in\Delta_1.$$

To substantiate the last equality in the two integrals of expression (2.26), we changed the integration variable $\nu = d-s.$ In addition, we see that the Goursat problem (2.22), (2.23) coincides with the Goursat problem (1.12), (1.13).

Similar to the solution (2.19) from formula (1.11) at $\varphi(x)=\varphi_1(x),\quad \psi(x)=\psi_1(x), x\in[0,d],$ $\mu(t)=\mu_2(t),\quad t\in[0,d_2],$ for $(\tilde{x},t)\in\tilde{\Delta}_2,$ we have the unique and stable classical solution

$$\begin{aligned}
\tilde{u}_2(\tilde{x},t) &= \frac{1}{2\tilde{a}(\tilde{x},t)}\Big((\tilde{a}\tilde{u}\tilde{v})(\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),0\},0)-\\
&-(\tilde{a}\tilde{u}\tilde{v})(\tilde{h}_2\{\tilde{g}_2(0,\tilde{h}^{(1)}[0,\tilde{g}_1(\tilde{x},t)]),0\},0)\Big)+\\
&+\frac{1}{2\tilde{a}(\tilde{x},t)}\int_{\tilde{h}_2\{\tilde{g}_2(0,\tilde{h}^{(1)}[0,\tilde{g}_1(\tilde{x},t)]),0\}}^{\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),0\}}[\tilde{\psi}_1(s)\tilde{v}(s,0)-\\
&-\tilde{\varphi}_1(s)\tilde{v}_\tau(s,0)+\tilde{b}(s,0)\tilde{\varphi}_1(s)\tilde{v}(s,0)]ds+\\
&+\frac{1}{2\tilde{a}(\tilde{x},t)}\int_0^t d\tau\int_{\tilde{h}_1\{\tilde{g}_1(\tilde{x},t),\tau\}}^{\tilde{h}_2\{\tilde{g}_2(\tilde{x},t),\tau\}}\tilde{\tilde{f}}_2(|s|,\tau)\tilde{v}(|s|,\tau)ds+\mu_2(t)-\\
&-\frac{1}{2\tilde{a}(0,t)}\int_0^t d\tau\int_{\tilde{h}_1\{\tilde{g}_1(0,t),\tau\}}^{\tilde{h}_2\{\tilde{g}_2(0,t),\tau\}}\tilde{\tilde{f}}_2(|s|,\tau)\tilde{v}(|s|,\tau)ds, (\tilde{x},t)\in\tilde{\Delta}_2.
\end{aligned} \tag{2.27}$$

Here the Riemann function $\tilde{v}(s,\tau)=\tilde{v}(s,\tau;x,t)$ is the classical solution to the Goursat problem of the form (1.14), (1.15). Using functions (2.16)–(2.18), we derive the following equalities:

$$\begin{aligned}
&(\tilde{a}\tilde{u}\tilde{v})(\tilde{h}_i\{\tilde{g}_i(0,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\},0)=\\
&=(auv)(d-\tilde{h}_i\{\tilde{g}_i(0,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\},0)=\\
&=(auv)(h_i\{\tilde{g}_i(0,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\},0)=\\
&=(auv)(h_i\{g_i(d,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\},0)=\\
&=(auv)(h_i\{g_i(d,h^{(j)}[d,\tilde{g}_j(\tilde{x},t)]),0\},0)=\\
&=(auv)(h_i\{g_i(d,h^{(j)}[d,g_j(x,t)]),0\},0),\\
&\tilde{h}_i\{\tilde{g}_i(0,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\}=\\
&=d-h_i\{\tilde{g}_i(0,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\}=\\
&=d-h_i\{g_i(d,\tilde{h}^{(j)}[0,\tilde{g}_j(\tilde{x},t)]),0\}=\\
&=d-h_i\{g_i(d,h^{(j)}[d,\tilde{g}_j(\tilde{x},t)]),0\}=\\
&=d-h_i\{g_i(d,h^{(j)}[d,g_j(x,t)]),0\},\\
&i\neq j, i, j=1,2.
\end{aligned} \tag{2.28}$$

In the solution (2.27) we apply the equalities (2.25), (2.28) and after a return replacement $\tilde{x}=d-x$ into the triangle Δ_3 becomes a solution

$$\begin{aligned}
u_3(x,t)&=\frac{1}{2a(x,t)}\Big((auv)(h_2\{g_2(x,t),0\},0)-\\
&-(auv)(h_2\{g_2(d,h^{(1)}[d,g_1(x,t)]),0\},0)\Big)+\\
&+\frac{1}{2a(x,t)}\int_{d-h_2\{g_2(d,h^{(1)}[d,g_1(x,t)]),0\}}^{d-h_2\{g_2(x,t),0\}}[\psi_1(d-s)v(d-s,0)-\\
&-\varphi_1(d-s)v_\tau(d-s,0)+\\
&+b(d-s,0)\varphi_1(d-s)v(d-s,0)]ds+\\
&+\frac{1}{2a(x,t)}\int_0^t d\tau\int_{d-h_1\{g_1(x,t),\tau\}}^{d-h_2\{g_2(x,t),\tau\}}\breve{f}_2(d-|s|,\tau)v(d-|s|,\tau)ds+\\
&+\mu_2(t)-\\
&-\frac{1}{2a(0,t)}\int_0^t d\tau\int_{d-h_1\{g_1(0,t),\tau\}}^{d-h_2\{g_2(0,t),\tau\}}\breve{f}_2(d-|s|,\tau)v(d-|s|,\tau)ds.
\end{aligned}$$

Here we carry out the reverse change of the integration variable $\rho=d-s$ and obtain the solution

$$\begin{aligned}
u_3(x,t)&=\frac{1}{2a(x,t)}\Big((auv)(h_2\{g_2(x,t),0\},0)-\\
&-(auv)(h_2\{g_2(d,h^{(1)}[d,g_1(x,t)]),0\},0)\Big)+\\
&+\frac{1}{2a(x,t)}\int_{h_2\{g_2(x,t),0\}}^{h_2\{g_2(d,h^{(1)}[d,g_1(x,t)]),0\}}[\psi_1(\rho)v(\rho,0)-\\
&-\varphi_1(\rho)v_\tau(\rho,0)+b(\rho,0)\varphi_1(\rho)v(\rho,0)]d\rho+\\
&+\frac{1}{2a(x,t)}\int_0^t d\tau\int_{d-h_1\{g_1(x,t),\tau\}}^{d-h_2\{g_2(x,t),\tau\}}\breve{f}_2(d-|s|,\tau)v(d-|s|,\tau)ds+\\
&+\mu_2(t)-\\
&-\frac{1}{2a(0,t)}\int_0^t d\tau\int_{d-h_1\{g_1(0,t),\tau\}}^{d-h_2\{g_2(0,t),\tau\}}\breve{f}_2(d-|s|,\tau)v(d-|s|,\tau)ds,
\end{aligned}$$

which coincides with the classical solution (2.11) at $k=n=1$ in triangle Δ_3 from theorem 2.1. Thus, the validity of Theorem 2.1 for mixed problem (2.1)–(2.3) on a rectangle Q_n is justified.

At the second step of mathematical induction, we assume that indicated in Theorem 2.1 the correctness criterion of the problem (2.1)–(2.3) and the formulas (2.9)–(2.11) for a unique and stable classical solution on Q_n are true and show that they are true on the rectangle Q_{n+1}. The mixed problem (2.1)–(2.3) in G_{n+1} for a function $u(x,t)$ by a nondegenerate change of variables $x=\hat{x},\quad t=\hat{t}+d_2$ is reduced for a function $\hat{u}(x,\hat{t})=u(x,\hat{t}+d_2)=u(x,t)$ in $\hat{G}_n$ to an equivalent mixed problem:

$$\begin{aligned}
&\hat{u}_{\hat{t}\hat{t}}(x,\hat{t})-\hat{a}^2(x,\hat{t})\hat{u}_{xx}(x,\hat{t})+\hat{b}(x,\hat{t})\hat{u}_{\hat{t}}(x,\hat{t})+\\
&+\hat{c}(x,\hat{t})\hat{u}_x(x,\hat{t})+\hat{q}(x,\hat{t})\hat{u}(x,\hat{t})=\\
&=\hat{f}(x,\hat{t}),(x,\hat{t})\in\hat{G}_n,
\end{aligned} \tag{2.29}$$

$$\hat{u}|_{\hat{t}=0}=\varphi_{n+1}(x),\ \hat{u}_{\hat{t}}|_{\hat{t}=0}=\psi_{n+1}(x),\ x\in[0,d], \tag{2.30}$$

$$\begin{aligned}
&\hat{u}|_{x=0}=\hat{\mu}_1(\hat{t}),\ \hat{u}|_{x=d}=\hat{\mu}_2(\hat{t}),\ \hat{t}\in[d_n,d_{n+1}],\\
&d_n=(n-1)h^{(2)}[d/2,g_2(0.0)],
\end{aligned} \tag{2.31}$$

with the coefficient

$$\hat{a}(x,\hat{t})=a(x,\hat{t}+d_2)=a(x,t),$$
$$\hat{b}(x,\hat{t})=b(x,\hat{t}+d_2)=b(x,t),$$

$$\hat{c}(x,\hat{t}) = c(x,\hat{t}+d_2) = c(x,t),$$
$$\hat{q}(x,\hat{t}) = q(x,\hat{t}+d_2) = q(x,t),$$

the right-hand side $\hat{f}(x,\hat{t}) = f(x,\hat{t}+d_2) = f(x,t)$ of the equation and boundary data

$$\hat{\mu}_i(\hat{t}) = \mu_i(\hat{t}+d_2) = \mu_i(t), \quad i=1,2.$$

After the non-degenerate replacement $x=\hat{x}$, $t=\hat{t}+d_2$, the characteristic functions $y_i = g_i(x,t)$ and their inverse functions $x = h_i\{y_i,t\}, t \ge 0$, $t = h^{(i)}[x,y_i], x \ge 0$, turn into functions:

$$\hat{y}_i = \hat{g}_i(x,\hat{t}) = g_i(x,\hat{t}+d_2) = g_i(x,t), \quad x,t \ge 0, i=1,2, \tag{2.32}$$

$$x = \hat{h}_i\{\hat{y}_i,\hat{t}\} = h_i\{\hat{y}_i,\hat{t}+d_2\} = h_i\{\hat{y}_i,t\}, \quad t \ge 0, i=1,2, \tag{2.33}$$

$$\hat{t} = \hat{h}^{(i)}[x,\hat{y}_i] = h^{(i)}[x,\hat{y}_i] - d_2, x \ge 0, i=1,2. \tag{2.34}$$

By the definition of inverse mappings, they satisfy the following inversion identities:

$$\hat{g}_i(\hat{h}_i\{\hat{y}_i,\hat{t}\},\hat{t}) = \hat{y}_i, \forall \hat{y}_i,$$
$$\hat{h}_i\{\hat{g}_i(x,\hat{t}),\hat{t}\} = x, x \ge 0, i=1,2,$$
$$\hat{g}_i(x,\hat{h}^{(i)}[x,\hat{y}_i]) = \hat{y}_i, \forall \hat{y}_i,$$
$$\hat{h}^{(i)}[x,\hat{g}_i(x,\hat{t})] = \hat{t}, \hat{t} \ge 0, i=1,2,$$
$$\hat{h}_i\{\hat{y}_i,\hat{h}^{(i)}[x,\hat{y}_i]\} = x, x \ge 0,$$
$$\hat{h}^{(i)}[\hat{h}_i\{\hat{y}_i,\hat{t}\},\hat{y}_i] = \hat{t}, \hat{t} \ge 0, i=1,2.$$

According to the hypothesis of mathematical induction, from formula (2.9) in Theorem 2.1 at $k=n$ and initial data $\varphi_{n+1}(x)$, $\psi_{n+1}(x), x \in [0,d]$, from the initial conditions (2.30)), in triangle $\hat{\Delta}_{3n-2}$ we find the unique and stable classical solution

$$\hat{u}_{3n-2}(x,\hat{t}) = \frac{1}{2\hat{a}(x,\hat{t})}\Big((\hat{a}\hat{u}\hat{v})(\hat{h}_2\{\hat{g}_2(x,\hat{t}),d_n\},d_n) + ((\hat{a}\hat{u}\hat{v})(\hat{h}_1\{\hat{g}_1(x,\hat{t}),d_n\},d_n)\Big) + \tag{2.35}$$
$$+\frac{1}{2\hat{a}(x,\hat{t})}\int_{\hat{h}_2\{\hat{g}_2(x,\hat{t}),d_n\}}^{\hat{h}_1\{\hat{g}_1(x,\hat{t}),d_n\}}[\psi_{n+1}(s)\hat{v}(s,d_n) - \varphi_{n+1}(s)\hat{v}_\tau(s,d_n) + \hat{b}(s,d_n)\varphi_{n+1}(s)\hat{v}(s,d_n)]ds +$$
$$+\frac{1}{2\hat{a}(x,\hat{t})}\int_{d_n}^{\hat{t}} d\tau \int_{\hat{h}_2\{\hat{g}_2(x,\hat{t}),\tau\}}^{\hat{h}_1\{\hat{g}_1(x,\hat{t}),\tau\}} \hat{f}(s,\tau)\hat{v}(s,\tau)ds, (x,\hat{t}) \in \hat{\Delta}_{3n-2}.$$

Applying the functions (2.32) (2.33), we derive the equalities

$$\hat{h}_i\{\hat{g}_i(x,\hat{t}),d_n\} = \hat{h}_i\{g_i(x,\hat{t}+d_2),d_n\} =$$
$$= h_i\{g_i(x,\hat{t}+d_2),d_n+d_2\} = h_i\{g_i(x,t),d_{n+1}\},$$
$$\hat{h}_i\{\hat{g}_i(x,\hat{t}),\tau\} = \hat{h}_i\{g_i(x,\hat{t}+d_2),\tau\} =$$
$$= h_i\{g_i(x,\hat{t}+d_2),\tau+d_2\} = h_i\{g_i(x,t),\tau+d_2\}, i=1,2,$$
$$(\hat{a}\hat{u}\hat{v})(\nu,d_n) = (auv)(\nu,d_n+d_2) = (auv)(\nu,d_{n+1}),$$
$$\hat{v}(\nu,d_n) = v(\nu,d_n+d_2) = v(\nu,d_{n+1}),$$
$$\hat{b}(\nu,d_n) = b(\nu,d_n+d_2) = b(\nu,d_{n+1}),$$

$$\int_{d_n}^{\hat{t}} d\tau \int_{\hat{h}_2\{\hat{g}_2(x,\hat{t}),\tau\}}^{\hat{h}_1\{\hat{g}_1(x,\hat{t}),\tau\}} \hat{f}(s,\tau)\hat{v}(s,\tau)ds =$$
$$= \int_{d_n}^{t-d_2} d\tau \int_{h_2\{g_2(x,t),\tau+d_2\}}^{h_1\{g_1(x,t),\tau+d_2\}} f(s,\tau+d_2)v(s,\tau+d_2)ds =$$
$$= \int_{d_{n+1}}^{t} d\varrho \int_{h_2\{g_2(x,t),\varrho\}}^{h_1\{g_1(x,t),\varrho\}} f(s,\rho)v(s,\varrho)ds, \tag{2.36}$$

where we applied the change of the integration variable $\varrho = \tau + d_2$. Hence we see that after the reverse change $\hat{t} = t - d_2$, solution (2.35) becomes solution (2.9) for $k=n+1$ in the triangle Δ_{3n+1}.

According to the hypothesis of mathematical induction in a similar way from formula (2.10) in Theorem 2.1 at $k=n$ and initial data $\varphi_{n+1}(x)$, $\psi_{n+1}(x), x \in [0,d]$, from the initial conditions (2.30), in triangle $\hat{\Delta}_{3n-1}$ we find the unique and stable classical solution

$$\hat{u}_{3n-1}(x,\hat{t}) = \frac{1}{2\hat{a}(x,\hat{t})}\Big((\hat{a}\hat{u}\hat{v})(\hat{h}_1\{\hat{g}_1(x,t),d_n\},d_n) - (\hat{a}\hat{u}\hat{v})(\hat{h}_1\{\hat{g}_1(0,\hat{h}^{(2)}[0,\hat{g}_2(x,\hat{t})]),d_n\},d_n)\Big) +$$
$$+\frac{1}{2\hat{a}(x,\hat{t})}\int_{\hat{h}_1\{\hat{g}_1(0,\hat{h}^{(2)}[0,\hat{g}_2(x,\hat{t})]),d_n\}}^{\hat{h}_1\{\hat{g}_1(x,\hat{t}),d_n\}}[\psi_{n+1}(s)\hat{v}(s,d_n) - \varphi_{n+1}(s)\hat{v}_\tau(s,d_n) + \hat{b}(s,d_n)\varphi_{n+1}(s)\hat{v}(s,d_n)]ds + \tag{2.37}$$
$$+\frac{1}{2\hat{a}(x,\hat{t})}\int_{d_n}^{\hat{t}} d\tau \int_{\hat{h}_2\{\hat{g}_2(x,\hat{t}),\tau\}}^{\hat{h}_1\{\hat{g}_1(x,\hat{t}),\tau\}} \hat{\breve{f}}_1(|s|,\tau)\hat{v}(|s|,\tau)ds + \hat{\mu}_1(\hat{t}) -$$
$$-\frac{1}{2\hat{a}(0,\hat{t})}\int_{d_n}^{\hat{t}} d\tau \int_{\hat{h}_2\{\hat{g}_2(0,\hat{t}),\tau\}}^{\hat{h}_1\{\hat{g}_1(0,t),\tau\}} \hat{\breve{f}}_1(|s|,\tau)\hat{v}(|s|,\tau)ds, (x,\hat{t}) \in \hat{\Delta}_{3n-1}.$$

Applying the functions (2.32)–(2.34), we come to the equalities

$$\hat{h}_i\{\hat{g}_i(0,\hat{h}^{(j)}[0,\hat{g}_j(x,\hat{t})]),d_n\} =$$
$$= \hat{h}_i\{\hat{g}_i(0,\hat{h}^{(j)}[0,g_j(x,\hat{t}+d_2)]),d_n\} =$$
$$= \hat{h}_i\{\hat{g}_i(0,h^{(j)}[0,g_j(x,t)] - d_2),d_n\} =$$
$$= \hat{h}_i\{g_i(0,h^{(j)}[0,g_j(x,t)] - d_2 + d_2),d_n\} =$$
$$= h_i\{g_i(0,h^{(j)}[0,g_j(x,t)]),d_n + d_2\} =$$
$$= h_i\{g_i(0,h^{(j)}[0,g_j(x,t)]),d_{n+1}\}, i \ne j, i,j=1,2,$$

$$\hat{\breve{f}}_p(|s|,\tau) = \breve{f}_p(|s|,\tau+d_2), \ p=1,2, \quad \hat{v}(|s|,\tau) = v(|s|,\tau+d_2). \tag{2.38}$$

Owing to the equalities (2.36), (2.38), by changing the variable $\hat{t} = t - d_2$, the classical solution (2.37) is transformed into the classical solution (2.10) for $k=n+1$ in the triangle Δ_{3n+2}.

Just like above, from formula (2.11) in Theorem 2.1 at $k=n$ and initial data $\varphi_{n+1}(x)$, $\psi_{n+1}(x)$ from the initial conditions (2.30) in triangle $\hat{\Delta}_{3n}$ we find the unique and stable classical solution

$$\hat{u}_{3n}(x,\hat{t}) = \frac{1}{2\hat{a}(x,\hat{t})}\Big((\hat{a}\hat{u}\hat{v})(\hat{h}_2\{\hat{g}_2(x,\hat{t}),d_n\},d_n) -$$
$$-(\hat{a}\hat{u}\hat{v})(\hat{h}_2\{\hat{g}_2(d,\hat{h}^{(1)}[d,\hat{g}_1(x,\hat{t})]),d_n\},d_n)\Big) +$$
$$+\frac{1}{2\hat{a}(x,\hat{t})}\int\limits_{\hat{h}_2\{\hat{g}_2(x,\hat{t}),d_n\}}^{\hat{h}_2\{\hat{g}_2(d,\hat{h}^{(1)}[d,\hat{g}_1(x,\hat{t})]),d_n\}}[\psi_{n+1}(s)\hat{v}(s,d_n) -$$
$$-\varphi_{n+1}(s)\hat{v}_\tau(s,d_n) + \hat{b}(s,d_n)\varphi_{n+1}(s)\hat{v}(s,d_n)]ds +$$
$$+\frac{1}{2\hat{a}(x,\hat{t})}\int\limits_{d_n}^{\hat{t}}d\tau\int\limits_{d-\hat{h}_1\{\hat{g}_1(x,\hat{t}),\tau\}}^{d-\hat{h}_2\{\hat{g}_2(x,\hat{t}),\tau\}}\hat{\breve{f}}_2(d-|s|,\tau)\hat{v}(d-|s|,\tau)ds +$$
$$+\hat{\mu}_2(\hat{t}) -$$
$$-\frac{1}{2\hat{a}(0,\hat{t})}\int\limits_{d_n}^{\hat{t}}d\tau\int\limits_{d-\hat{h}_1\{\hat{g}_1(0,\hat{t}),\tau\}}^{d-\hat{h}_2\{g_2(0,\hat{t}),\tau\}}\hat{\breve{f}}_2(d-|s|,\tau)\hat{v}(d-|s|,\tau)ds,$$
$$(x,\hat{t}) \in \hat{\Delta}_{3n}. \qquad (2.39)$$

Using the functions (2.32)–(2.34), we find the equalities

$$\hat{h}_i\{\hat{g}_i(d,\hat{h}^{(j)}[d,\hat{g}_j(x,\hat{t})]),d_n\} =$$
$$= \hat{h}_i\{\hat{g}_i(d,\hat{h}^{(j)}[d,g_j(x,\hat{t}+d_2)]),d_n\} =$$
$$= \hat{h}_i\{\hat{g}_i(d,h^{(j)}[d,g_j(x,t)]-d_2),d_n\} =$$
$$= \hat{h}_i\{g_i(d,h^{(j)}[d,g_j(x,t)]-d_2+d_2),d_n\} =$$
$$= h_i\{g_i(d,h^{(j)}[d,g_j(x,t)]),d_n+d_2\} =$$
$$= h_i\{g_i(d,h^{(j)}[d,g_j(x,t)]),d_{n+1}\},\ i \neq j,\ i,j = 1,2.$$
$$\int\limits_{d_n}^{\hat{t}}d\tau\int\limits_{d-\hat{h}_1\{\hat{g}_1(x,\hat{t}),\tau\}}^{d-\hat{h}_2\{\hat{g}_2(x,\hat{t}),\tau\}}\hat{\breve{f}}_2(d-|s|,\tau)\hat{v}(d-|s|,\tau)ds =$$
$$= \int\limits_{d_n}^{t-d_2}d\tau\int\limits_{d-h_1\{g_1(x,t),\tau+d_2\}}^{d-h_2\{g_2(x,t),\tau+d_2\}}\breve{f}_2(d-|s|,\tau+d_2)\times$$
$$\times v(d-|s|,\tau+d_2)ds =$$
$$= \int\limits_{d_{n+1}}^{t}d\delta\int\limits_{d-h_1\{g_1(x,t),\delta\}}^{d-h_2\{g_2(x,t),\delta\}}\breve{f}_2(d-|s|,\delta)v(d-|s|,\delta)ds, \quad (2.40)$$

where we have implemented the replacement of the integration variable $\delta = \tau + d_2$.

Now it is easy to make sure with the help of equalities (2.36), (2.38), (2.40), that by inverse replacement $\hat{t} = t - d_2$ the solution (2.39) becomes a solution $u_{3n+3}(x,t)$ of the form (2.11) at $k = n+1$ in the triangle Δ_{3n+3}.

So, the validity of formulas (2.9)–(2.11) in triangles Δ_{3k-l}, $l = 0,1,2$, $k = \overline{1,n}$, of the classical solution to the mixed problem (2.1)–(2.3) is substantiated by the method of mathematical intuition. It remains to prove a correctness criterion of this mixed problem.

By the assumption of the method of mathematical induction on the rectangle Q_n, the Hadamard correctness criterion for the mixed problem (2.1)–(2.3) on the rectangle $\hat{G}_n$ the following smoothness requirements (2.4), (2.7), (2.8) at $k = n$ from theorem 2.1:

$$\varphi \in C^2[0,d],\ \psi \in C^1[0,d],$$
$$\hat{\mu}_1,\hat{\mu}_2 \in C^2[d_n,d_{n+1}],\ \hat{f} \in C(\hat{G}_n), \qquad (2.41)$$

$$\int\limits_{d_n}^{\hat{t}}\hat{f}\left(|\hat{h}_i\{\hat{g}_i(x,\hat{t}),\tau\}|,\tau\right)d\tau \in C^1(\hat{\Delta}_{3n-2}\bigcup\hat{\Delta}_{3n-1}), \qquad (2.42)$$
$$i = 1,2,$$

$$\int\limits_{d_n}^{\hat{t}}\hat{f}\left(d-|d-\hat{h}_i\{\hat{g}_i(x,\hat{t}),\tau\}|,\tau\right)d\tau \in C^1(\hat{\Delta}_{3n-2}\cup\Delta_{3n}),$$
$$i = 1,2. \qquad (2.43)$$

These smoothness grants twice continuous differentiability of the solution to the auxiliary mixed problem (2.29)–(2.31) in the triangles $\hat{\Delta}_{3n-2}$, $\hat{\Delta}_{3n-1}$, $\hat{\Delta}_{3n}$. Since after the inverse change $\hat{t} = t - d_2$, the corresponding transformations from (2.36), (2.38), (2.40) to the smoothness requirements (2.41)–(2.43) become the smoothness requirements (2.4), (2.7), (2.8) at $k = n+1$ from Theorem 2.1, then these smoothness requirements from Theorem 2.1 are equivalent to twice continuous differentiability of the solution to the mixed problem (2.1)–(2.3) in the triangles Δ_{3n+1}, Δ_{3n+2}, Δ_{3n+3}. In view of the assumption of mathematical induction, the smoothness requirement (2.41)–(2.43) together with the matching conditions (2.5), (2.6) from Theorem 2.1 gives twice continuous differentiability of a solution to the auxiliary mixed problem (2.29)–(2.31) on the characteristics $\hat{g}_2(x,\hat{t}) = \hat{g}_2(0,d_2)$, $\hat{g}_1(x,\hat{t}) = \hat{g}_1(d,d_2)$ of the equation (2.29) in $\hat{G}_n$. Thus, by the assumption of mathematical induction, a solution to the problem (2.29)–(2.31) is twice continuous differentiable on $\hat{G}_n$.

Due to the fact that above the functions u_{3n+1}, u_{3n+2}, u_{3n+3} were derived by us from the functions $\hat{u}_{3n-2}$, $\hat{u}_{3n-1}$, $\hat{u}_{3n}$ by the non-degenerate replacement $\hat{x} = x$, $\hat{t} = t - d_2$ then, therefore, these functions u_{3n+1}, u_{3n+2}, u_{3n+3} are twice continuous differentiable everywhere in G_{n+1} and, in particular, on the characteristics $g_2(x,t) = g_2(0,d_{n+1})$, $g_1(x,t) = g_1(d,d_{n+1})$ of the equation (2.1) in G_{n+1}, since by construction the recurrent initial data from (2.12) is $\varphi_{n+1}(x)$, $\psi_{n+1}(x) \in C^2[0,d]$. Therefore, we still need to show twice continuous differentiability of the solutions $u \in C^2(G_{n+1})$ and $u \in C^2(Q_n)$ on the common side $t = d_{n+1}$ of these rectangles G_{n+1} and Q_n.

By construction, on the common side $t = d_{n+1}$ of these rectangles, the equalities are true

$$u_{3n-j}(x,d_{n+1}) = \varphi_{n+1}(x) = u_{3n+1}(x,d_{n+1}),$$

$$\frac{\partial u_{3n-j}(x,t)}{\partial t}\Big|_{t=d_{n+1}} = \psi_{n+1}(x) = \frac{\partial u_{3n+1}(x,t)}{\partial t}\Big|_{t=d_{n+1}} \quad (2.44)$$

for all $x \in [jd/2,(j+1)d/2]$, $j = 0,1$, from recurrent initial data (2.12). Differentiating equalities (2.44) once and twice with respect to x and t using the equation (2.1), we calculate the values of the partial derivatives:

$$\frac{\partial u_{3n-j}(x,d_{n+1})}{\partial x} = \varphi'_{n+1}(x) = \frac{\partial u_{3n+1}(x,d_{n+1})}{\partial x},$$

$$\frac{\partial^2 u_{3n-j}(x,d_{n+1})}{\partial x^2} = \varphi''_{n+1}(x) = \frac{\partial^2 u_{3n+1}(x,d_{n+1})}{\partial x^2},$$

$$\frac{\partial^2 u_{3n-j}(x,t)}{\partial x \partial t}\Big|_{t=d_{n+1}} = \psi'_{n+1}(x) = \frac{\partial^2 u_{3n+1}(x,t)}{\partial x \partial t}\Big|_{t=d_{n+1}},$$

$$\frac{\partial^2 u_{3n-j}(x,t)}{\partial t^2}\Big|_{t=d_{n+1}} = f(x,d_{n+1}) + a^2(x,d_{n+1})\varphi''_{n+1}(x) - b(x,d_{n+1})\psi'_{n+1}(x) - c(x,d_{n+1})\varphi'_{n+1}(x) - q(x,d_{n+1})\varphi_{n+1}(x) = \frac{\partial^2 u_{3n+1}(x,t)}{\partial t^2}\Big|_{t=d_{n+1}},\ x \in \left[j\frac{d}{2},(j+1)\frac{d}{2} \right],\ j = 0,1.$$

Here, when deriving the last equality, we used all the previous equalities. These equalities imply twice continuous differentiability of functions (2.9)–(2.11) for $k = \overline{1,n}$ and $k = n+1$ at the intersection $t = d_{n+1}$ of rectangles G_{n+1} and Q_n. Theorem 2.1 is proved.

***Corollary* 2.1.** *If the right-hand side f depends only on x and is continuous in x, that is* $f \in C[0,d]$, *or depends only on t and is continuous in t, that is* $f \in C[0,d_{n+1}]$, *then the assertion of this theorem* 2.1 *is true without integral smoothness requirements* (2.7), (2.8).

When the function *f* depends only on *x* or *t* and is continuous in Q_n, then the integral requirements (2.7), (2.8) in the theorem 2.1 are automatically satisfied.

***Corollary* 2.2.** *Let the coefficients of the equation* (2.1) *be* $a(x,t) \geq a_0 > 0, (x,t) \in Q_n$, $a \in C^2(Q_n)$, $b,c,q \in C^1(Q_n)$. *If the right-hand side f depends on x and t, then in the smoothness requirements* (2.7), (2.8) *the belonging of integrals to the sets, respectively,* $C^1(\Delta_{3k-2} \bigcup \Delta_{3k-1})$ *and* $C^1(\Delta_{3k-2} \bigcup \Delta_{3k})$ *are equivalent to their belonging to sets, respectively,* $C^{(0,1)}(\Delta_{3k-2} \bigcup \Delta_{3k-1})$ *and* $C^{(0,1)}(\Delta_{3k-2} \bigcup \Delta_{3k})$ *or* $C^{(1,0)}(\Delta_{3k-2} \bigcup \Delta_{3k-1})$ *and* $C^{(1,0)}(\Delta_{3k-2} \bigcup \Delta_{3k})$, $k = \overline{1,n}$. *Here* $C^{(0,1)}(\Omega)$ $(C^{(0,1)}(\Omega))$ *is the set of all continuous* (*continuously differentiable*) *with respect to x and continuously differentiable* (*continuous*) *with respect to t functions on a set* Ω.

Corollary 2.2 is proved in the same way as for constant coefficients $a(x,t) = const$, $b = c = q = 0$ in the candidate dissertation [16].

***Remark* 2.1.** The global theorem to the Hadamard correctness of the first mixed problem for inhomogeneous model telegraph equation (2.1) with variable coefficients

$$a(x,t) \neq const,\ b = -a_t(x,t)/a(x,t),$$
$$c = -a(x,t)a_x(x,t),\ q = 0$$

in a rectangle Q_n is proved in author's articles [2], [3].

Conclusion

The global Theorem 2.1 to Hadamard correctness of the first mixed problem is proved for inhomoge-neous general telegraph equation (2.1) with variable coefficients in a rectangle Q_n which, in the limit at $n \to +\infty$, exhausts a half-strip $G = [0,d] \times [0,+\infty[$. Without explicitly continuing the problem data $f, \varphi, \psi, \mu_1, \mu_2$ outside an assignment set Q_n for the first mixed problem (2.1)–(2.3) in $\dot{Q}_n$, explicit recurrent Riemann-type formulas (2.9)–(2.11) of a unique and stable classical solution $u \in C^2(Q_n)$ are derived by a novel method of auxiliary mixed problems. The correctness criterion is established as inclusions (2.4)–(2.8). The smoothness requirements (2.4), (2,7), (2.8) are necessary and sufficient for twice continuous differentiability of functions (2.9)–(2.11) in triangles Δ_{3k-l}, $l = 0,1,2$, $k = \overline{1,n}$. Six matching conditions (2.5), (2.6) together with the smoothness requirements (2.4), (2,7), (2.8) are necessary and sufficient for twice continuous differentiability of the solution (2.9)–(2.11) on characteristics $g_2(x,t) = g_2(0,d_k)$, $g_1(x,t) = g_1(d,d_k)$, $k = \overline{1,n}$, into Q_n.

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